

# Mining Market Directions: A Type of Trading Strategy for Trend Following and Mean Reverting Index

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**Abstract:** Investors generally go long when the market is rising and short when the market is falling. In order to implement trading strategies more effectively, it is very important to mine the market direction and the timing of long and short positions. This article divides the direction of the market into three categories: Random Walk (RW), Trend Following (TF) caused by negative reflection, and Mean Reverting (MR) caused by overreaction. Push-Response Test, Variance Ratio Test, and P+, P-Test are used to determine the direction of the market, and finally formulate specific trading strategies according to the market type to achieve profitability. This article summarizes the market of certain indexes of US futures, such as HO, GC, etc., which have TF trends in recent years. Taking the ES (E-mini S&P 500) contract as an example, the results show that ES has a trend of TF in the last 6 years, and a trend of MR in 97-09. This article proposes quantitative strategies based on TF and MR market respectively, and applies it to the ES index to verify the effectiveness of the trading strategies.

## 1. Introduction

Since the birth of the market, investors have been keen to trade stocks and derivatives in the market, but it is very difficult to develop an accurate and effective specific trading strategy. In 1987, Andrew Lo et. al. found that stock prices do not follow random walk [1]. In 2003, Farmer J D et al. found that effective markets have long-term memory, and if the long-term nature is not considered, the analysis may be invalid [2-3]. In 2005, Alexei Chekhlov et al. proposed an indicator to measure the quality of transactions [4-5]. In 2006, Bouchaud J P et al. stated that some markets are considered to follow a power law distribution and have volatility clustering [6-8]. In 2012, NN Taleb et al. proposed that the market has fat tail characteristics [9-10].

Based on the regularization data of the HO, GC, ES and other indexes of the US futures market in the past two decades, this article analyzes the market characteristics of different time periods through three testing methods, and derives trading strategies for different time periods.

## 2. Analysis of the original price series

### 2.1 Stabilization of the price series

This section takes the ES index as an example. The trading time of ES is 8:30 am-3:15 pm (Chicago time), totaling 6 hours and 45 minutes, i.e. 405 minutes. According to the ES data with the frequency of 1 minute from 1997 to 2014, an image of daily volatility is obtained. Due to the short time interval,  $\Delta P$  is used to approximate volatility.

Figure 1 shows that the volatility is high in the previous hour, and the middle part slowly decrease until it is close to the closing time. This is consistent with the actual situation. At the opening of the day, the information from last night floods into the market, which affects the price to a higher extent. However, because there is no new information in Japan and China, investors are waiting and watching, resulting in relatively small changes in stock prices. Some investors begin to speculate about changes in the fundamentals of each stock after the market closed, so trading volume rose, resulting in higher price changes than the day. Eliminate some external influences and determine whether the stock price is predictable, so that better analysis strategies are obtained. Consider dividing

the 405 minutes of the trading day into 7 sub-segments, the first 6 sub-segments are all one hour apart, and the corresponding volatility is removed respectively to standardize the stock price sequence to eliminate intraday seasonality.

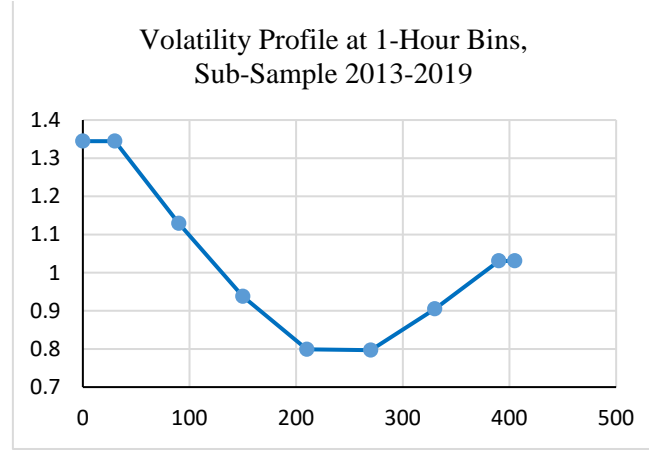


Fig. 1. Volatility Profile at 1-Hour Bins, Sub-Sample 2013-2019

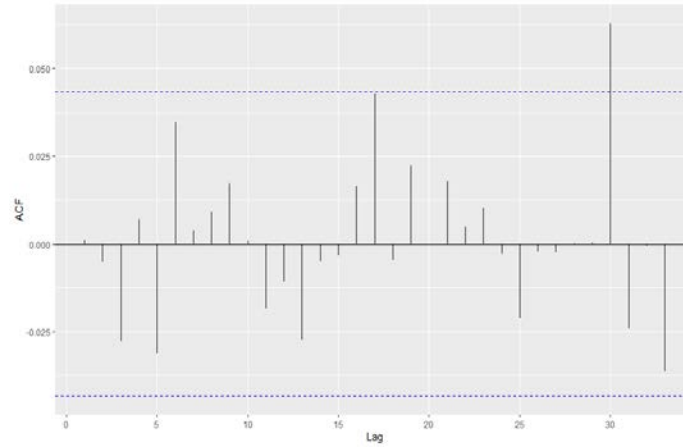


Fig. 2. Autocorrelation Function of ES Closing Price After Standardization

For a stationary random process, the Fourier transform of the autocorrelation function must be non-negative; conversely, for a random process, if the Fourier transform of the autocorrelation function is non-negative, the stationarity is obtained.

## 2.2 Predictability of the price series

Figure 2 is the image of the autocorrelation function after eliminating the intraday seasonality. It can be inferred that the standardized ES closing price tends to be stable. Statistics show that the logarithmic rate of return represents a certain degree of heavy tailing compared to the normal distribution. Therefore, the Levy distribution is used as the parametric model of the prior distribution of log returns, and the Kolmogorov test proves the validity of the model. It can prove that the attenuation of the autocorrelation function under the Levy distribution hypothesis presents a power law, which belongs to long-term memory, and also indicates the predictability of the data set.

## 3. Testing the trend of price series

### 3.1 Push-Response Test

Push-Response Test considers the impact of the previous price change on current price change to detect market trends. X-axis is the percentage of the last price change, and y-axis is the percentage of the current price change. For various time separation periods  $\tau$ , after removing the extreme points, graph and connect them with polylines. The part of the line chart falling in the first and third quadrants indicates that the market has a TF trend, because the last price change direction and the current price

change direction are the same. Similarly, the parts falling in the second and fourth quadrants can indicate that when the market changes by those specific percentages, there will be a MR trend.

### 3.2 Variance ratio test

Assume that the closing price time series  $p(t)$  is stable:  $\langle p(t) \rangle = \mu$ ,  $\langle p(t_1), p(t_2) \rangle = R(t_1, t_2)$ , and  $R(t_1, t_2)$  is the function  $R(\tau)$  of  $\tau = t_1 - t_2$ . Let  $\Delta p_k = p(t + k\tau) - p[t + (k - 1)\tau]$ ,  $\rho_k = \text{Corr}(\Delta p_t, \Delta p_{t+k})$  and  $\text{Var}(q) = \text{Var}(\sum_{i=1}^q \Delta p_i)$ .

Construct the statistics:

$$VR(q) = \frac{\text{Var}(q)}{q \cdot \text{Var}(1)} = \frac{q \cdot \text{Var}(1) + 2q \cdot \text{Var}(1) \cdot \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right) \rho_k}{q \cdot \text{Var}(1)} = 1 + 2 \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right) \rho_k.$$

The variance of a random walk is proportional to time  $q$ , and the standard deviation is proportional to  $\sqrt{q}$ . Therefore, if the variance ratio of the ES index remains unchanged during a certain time period, the market will move randomly during this period. From the above formula, the variance ratio can be decomposed into the sum of the autocorrelation functions with positive coefficients. In a word, if the trend of  $VR(q)$  is increasing then the market is TF, the decreasing is MR, and the level is RW [11].

An example of random walk is the log-Brownian process:  $\frac{dP}{P} = \mu dt + \sigma dW$ . Discretize it in the following way:  $P_{t+1} = P_t \left\{ 1 + \frac{\mu}{253} + \frac{\sigma}{\sqrt{253}} \cdot \varepsilon_i \right\}$ . An example of mean reversion is the log-Ornstein-Uhlenbeck process:  $\frac{dP}{P} = -a(P - \bar{P})dt + \sigma dW$ , where  $\bar{P}$  is the mean value of the index over a period of time. Discretize in a similar way:  $P_{t+1} = P_t \left\{ 1 - a \cdot (P - \bar{P}) + \frac{\sigma}{\sqrt{253}} \cdot \varepsilon_i \right\}$ .

Therefore, by comparing the slope near the small time offset  $q$ , it is possible to distinguish sequence trend following, mean reverting and random walk.

### 3.3 $P_+$ , $P_-$ Test

#### 3.3.1 The first type of market trends testing method

In the first testing method, the following variables are defined:  $\tau$  is time separation period,  $P_+(\tau)$  is the number of pairs of consecutive price change in the same direction and  $P_-(\tau)$  is the number of pairs of consecutive price change in the opposite direction. Thus the probability of  $P_+(\tau)$  is  $p^2 + q^2$  and the probability of  $P_-(\tau)$  is  $2pq$ , where  $p$  is the probability of the index raise and  $q$  is the probability of the index decline during each time separation period  $\tau$ .

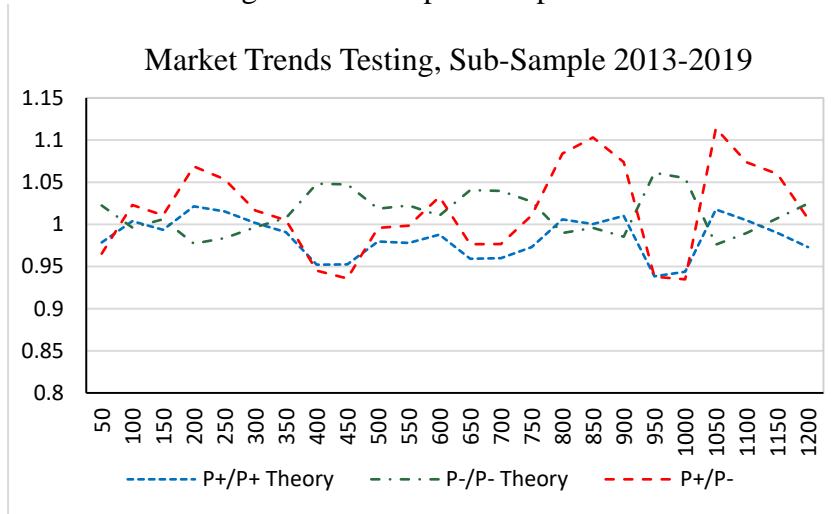


Fig. 3. Market Trends Testing, Sub-Sample 2013-2019

Figure 3 plots the changes of the three variables  $P_+/P_+$  Theory,  $P_-/P_-$  Theory and  $P_+/P_-$  as the time separation interval  $\tau$  increases. In general, the parts of the image where the three variables are

greater than 1 indicate that the ES index has the characteristics of TF. The reason is that after a price change, the price of the next time interval is more likely to change in the same direction. In the same way, the part of the image less than 1 indicates that the price in the next time interval is more likely to move in the opposite direction, that is, the market tends to be more MR.

Figure 3 can lead to three conclusions: First, almost no time period in the ES index is RW. Second, the ES index has a trend of MR in the short-term, and reflects the trend of TF in the medium and long-term. Third, the characteristics of TF and MR of the ES index are produced alternately. In other words, after a period of TF (MR), the next period of time is more likely to be MR (TF).

### 3.3.2 The second type of market trends testing method

The second type testing method has similar idea with the first type testing method. The following variables are defined:  $M$  is the time series length,  $L$  is the chain length,  $P_+(\tau, l)$  is the number of chains of consecutive price change of length  $l$  in the positive direction and  $P_-(\tau, l)$  is the number of chains of consecutive price change of length  $l$  in the negative direction. Therefore,  $P_+(\tau, l) = (M - L + 1)p^L$  and  $P_-(\tau, l) = (M - L + 1)q^L$ .

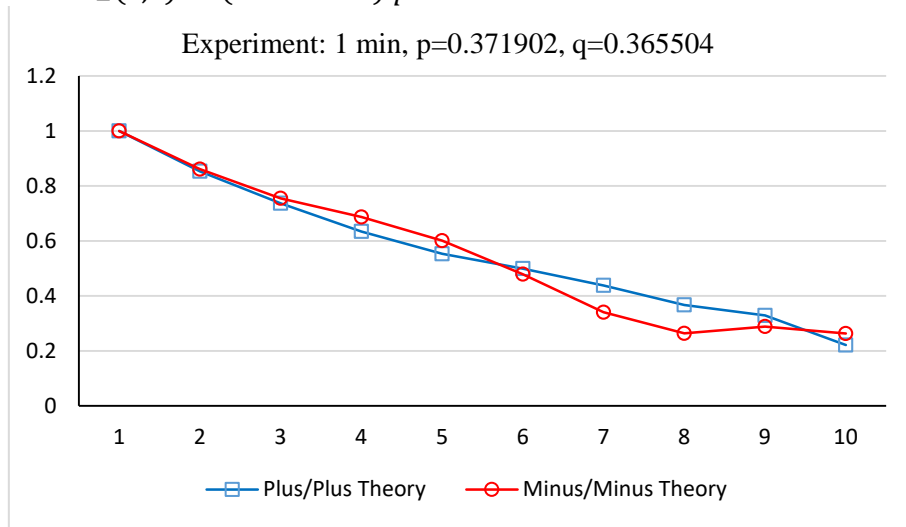


Fig. 4. Experiment: 1 min, p=0.371902, q=0.365504

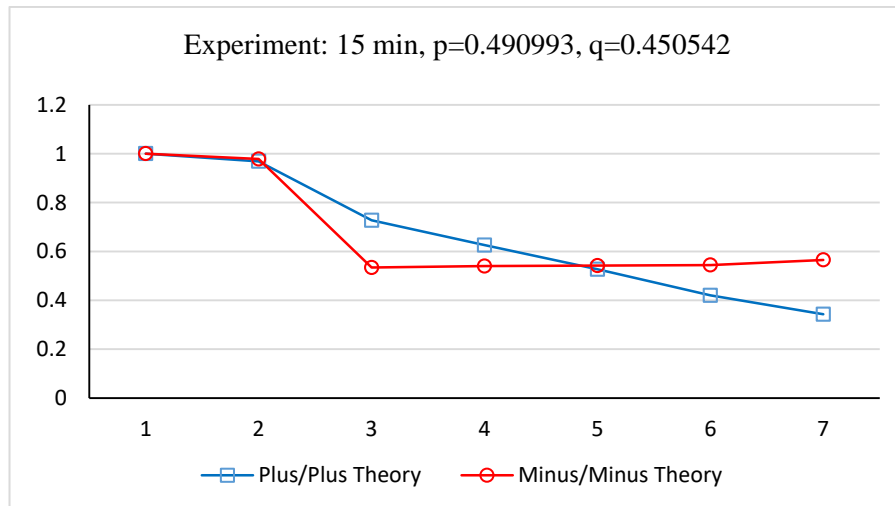


Fig. 5. Experiment: 15 min, p=0.490993, q=0.450542

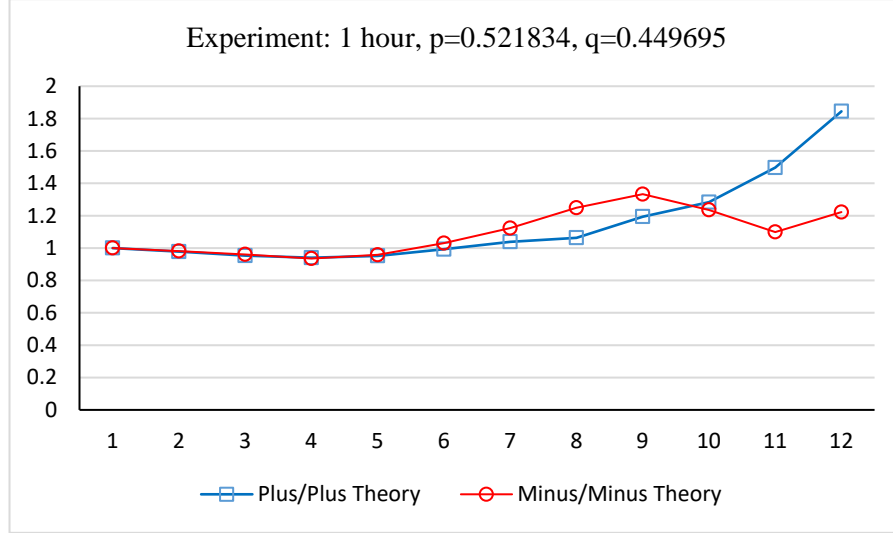


Fig. 6. Experiment: 1 hour, p=0.521834, q=0.449695

Figure 4-6 shows the ratio of the actual value and the theoretical value of continuous price changes in the same direction when the time separation periods are 1min, 15min and 60min respectively. The horizontal axis represents the number of consecutive changes in the same direction. *Plus* means continuous increase, and *Minus* means continuous decrease. It should be noted that the second type testing method ignores the impact of reverse changes in prices.

In general, if Plus/Plus Theory is greater than 1, it means that after the ES index continues to rise, there is a possibility that it will continue to rise higher than the theoretical value. In the same way, if Minus/Minus Theory is greater than 1, it means that after the ES index continues to decline, there is a possibility that it will continue to decline even greater than the theoretical value. Both results indicate the phenomenon of TF. If Plus/Plus Theory and Minus/Minus Theory are less than 1, it means that after the index continues to rise or go downward, it is more likely to turn down or go upward at the next price change. This also reflects the MR trend to a certain extent.

It can be inferred from Figure 4-6 that at  $\tau = 1$  and  $\tau = 15$ , the ES index has significant MR characteristics, and when  $\tau = 60$ , the performance of the ES index is more inclined to TF. As  $\tau$  increases, the characteristics of MR become weaker and weaker, while the characteristics of TF are slowly increasing. This is consistent with the previous test results.

#### 4. Trading strategies

After analyzing the market trend, this section will propose two low-frequency trading strategies. The time separation period of the data set is 5 minutes, that is,  $\tau = 5$ .

The data set collects Open Price, Close Price, High Price and Low Price for every five minutes. All possible trading times are when a new closing price occurs, no trading will take place within a 5-minute interval. Suppose that the initial position is 0 and our strategies consider the transaction cost.

##### 4.1 Trend following strategy

First, introduce some parameters and concepts to help to explain the model. Assuming that three parameters  $\delta$ ,  $\alpha$  and  $\beta$  need to be estimated, *HH* (*Highest\_High*) is the maximum value of high price of  $\delta$  consecutive 5 minutes intervals, *LL* (*Lowest\_Low*) is the minimum value of low price of  $\delta$  consecutive 5 minutes intervals and *Entry\_Price* is the price of entering the position.

Assuming that the number of positions is at most 1 contract, thus the cost of the strategy will be relatively low. The mechanism of this trading strategy is as follows: since the market is following a TF trend and the position is 0 at the beginning, when the index touches  $(1 - \beta) * HH$  in a 5 minutes interval, go long a contract, and when the index touches  $(1 - \beta) * LL$ , short sale a contract. When the position is +1, set the Entry Price to Close Price if the Close Price is greater than the Entry Price for every five-minute interval, therefore, the Entry Price can be approximately considered as *HH*. If the

market shows a small MR trend, the investor is confident to be the long position. However, if the market has a relatively large MR phenomenon, continuing to hold the position may cause a large loss, so there is a reason to close the position at this time and wait for the next investment opportunity. This strategy uses  $\alpha$  to measure the degree of MR and will close the position when the index reaches  $Entry\ Price * (1 - \alpha)$ . Similarly, when the position is -1, this strategy is going to obtain profit when the market continue to go downward. Therefore, if the market recovers to a certain extent, this strategy will close the position. As a result, this strategy will close the position when the index reaches  $Entry\ Price (1 + \alpha)$ .

There are some special cases: since the Open Price of the 5 minutes is not necessarily equal to the Close Price of the last time, in some cases, the Open Price may directly pass through *HH* or *LL*. This strategy will take the maximum value of *HH* and the Open Price and the minimum value of *LL* and the Open Price to calculate the profit. In addition, no matter what the situation is, as long as the price reaches *HH* or *LL*, the position will be directly changed to +1 (-1).

Using the ES index data from 2013 to 2016 to train the strategy and 2017 to 2019 to backtest the strategy. Through the above analysis, it has been obtained that the 1-minute data of ES index has a small MR trend in the short term and a TF trend in the medium and long term. Because the strategy uses 5-minute data, the trend of TF will be more obvious than 1-minute data. *Net Profit*, *Worst Draw Down*, *Sharpe Ratio* etc. indicators are selected to measure the performance of the strategy.

## 4.2 Mean-reverting strategy

If the long-term market shows the trend of MR, the above trading strategy will produce a very bad result. Therefore, a new strategy needs to be proposed. In the MR strategy, five parameters  $\delta, \gamma, \alpha, N1, N2$  need to be trained. Introduce two new concepts: MA (Mean Average) is the average of all Close Prices of  $\delta$  consecutive 5-minute intervals and Vol (Volatility) is calculated based on the data of  $\gamma$  consecutive 5-minute intervals.

Assume that the maximum number of positions held is 2. The new strategy will be more complicated than the TF strategy. The trading strategy's mechanism is as follows: when the position  $> 0$ , this strategy will close all the positions in two situations: when the index reaches MA or when the index reaches  $Entry\ Price(1 - \alpha)$ . The Entry Price is the average of all positions (1 or 2). When the index reaches the MA, due to the characteristics of MR, it will be difficult for investors to predict the direction of the next moving, so the safest way is offset. When the position is less than 0, similar to the above situation, this strategy will be offset when the index reaches MA or when the index reaches  $Entry\ Price (1 + \alpha)$ . If one of the above four conditions is met during the transaction, this strategy will change the trading order to false at the time of the offset until it is confirmed that the market at the moment has the characteristics of MR. Only at least one of the three conditions:  $Close\ Price < MA$ ,  $High > MA$ , or  $Close\ Price > MA$ ,  $Low < MA$  or  $High > MA$ ,  $Low < MA$  is met, the transaction order will be changed to true. After the trading order is changed to true, this strategy will go long when the index reaches  $MA * (1 - N1 * Vol)$  and short sale at  $MA * (1 + N1 * Vol)$ . If the market continues to fall or rise, investors have sufficient confidence to believe that the market will rebound, as a result, the strategy will go long the second contract at  $MA * (1 - (N1 + N2) * Vol)$ . Short sell the second contract at  $MA * (1 + (N1 + N2) * Vol)$ . In order to control risks, this strategy will no longer make the third transaction. Similar to the TF strategy, when special circumstances occur, the best solution is to consider the maximum or the minimum, but the MR strategy will never directly establish the opposite direction of the position. The above analysis shows that the 1997-2003 ES index data are showing MR trend. Utilize these data to train the parameters, and the 2004-2009 data for testing.

### 4.3 Strategy backtest results

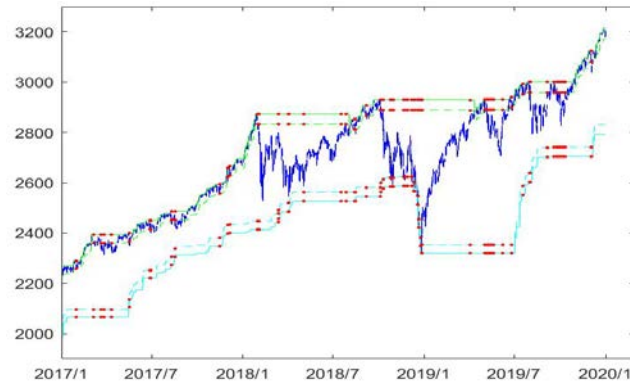


Fig. 7. TF Trading Strategy Backtesting Result

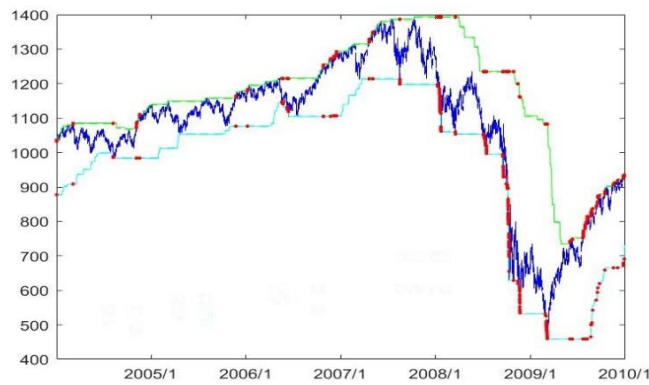


Fig. 8. MR Trading Strategy Backtesting Result

Table 1 TF and MR Trading Strategy Backtesting Indicators Result

Index	Value	Index	Value
Net Profit	83762.4	Net Profit	10261.65
WorstDrawDown	-9235.975	WorstDrawDown	-29426.2625
NPTWDD	906.9145 %	NPTWDD	34.8724 %
trades	74	Trades	154
Avg Rate Of Return	27.7374 %	Avg Rate Of Return	1.6406 %
Standard Deviation	9.8658 %	Standard Deviation	11.2781 %
Sharpe Ratio	2.8115	Sharpe Ratio	0.14547

Figure 7-8 and Table 1 shows the TF and MR trading strategy backtesting result. The red dots indicate the trading time, the green line and the blue line indicate the  $HH$  and  $LL$ . The image shows that both of the two strategies capture the trends of the ES index successfully. TF strategy has a good performance on the ES index. The average annual return is greater than 27%, the Sharpe ratio has reached 281%, and NPTWDD (Net Profit / Worst Draw Down) is more than 900%, which shows that TF strategy has high return and low risk. MR strategy's performance of various indicators is not as good as TF strategy, because the trend of MR is only reflected in the short-term, and this trading strategy is a long-term strategy. However, MR strategy is also profitable. The test shows that the  $\alpha$  and  $\beta$  of TF strategy are significantly larger than the  $\alpha$  and  $\beta$  of MR strategy, which also shows that TF strategy tends to long-term holding, while MR strategy tends to frequently change positions.

### 5. Conclusion

This article introduces four methods to test market directions, namely Push Response Test, Variance Raito Test and two kinds of  $P+$  and  $P-$  Test. Taking ES index as a representative, two kinds of  $P+$

and P— Test are used to test the trend of ES index. It is worth mentioning that there are many indexes in the US futures market (such as HO, GC, etc.), similarly to the ES index has recently shown the trend of TF. In addition, this article proposes two quantitative trading strategies for TF and MR markets respectively, and verified the effectiveness of the trading strategies based on the backtesting and indicators data.

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